Holographic RG flows in six dimensional F(4) gauged supergravity

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Abstract

We study critical points of $F(4)$ gauged supergravity in six dimensions coupled to three vector multiplets. Scalar fields are described by $\mathbb{R}^+ \times \frac{SO(4,3)}{SO(4)\times SO(3)}$ coset space, and the gauge group is given by $SO(3)_R \times SO(3)$ with $SO(3)_R$ being the R-symmetry. We identify new non-supersymmetric critical points of the scalar potential. One of these new critical points is shown to be stable with all scalar masses are above the BF bound and should correspond to a new non-supersymmetric CFT in five dimensions. On the other hand, the maximally supersymmetric critical point with all scalars vanishing is dual to an SCFT_5 arising from a near horizon geometry of the D4-D8 brane system in type I′ theory with a global symmetry $E_1 \sim SU(2)$. We give a numerical RG flow solution interpolating between this SCFT and the new critical point. The flow describes a non-supersymmetric deformation driven by relevant operators of dimension 3. We identify the dual operators with the mass terms for hypermultiplet scalars in the dual field theory. The solution provides another example of holographic RG flows in $\mathrm{AdS}_6/\mathrm{CFT}_5$ correspondence.

1 Introduction

The study of holographic renormalization group (RG) flow is one of the most important applications of the AdS/CFT correspondence [1]. Soon after the original proposal of the correspondence, many works considering RG flows in five dimensional gauged supergravity have been done, see for example [2], [3] and [4]. These results describe various perturbations of $N = 4$ SYM in four dimensions. Since the AdS/CFT correspondence has been extended to other dimensions not only for the duality between an AdS_5 supergravity and a CFT$_4$ [5], [6], [7], [8] and [9], it is interesting to study holographic RG flows in these extensions as well. Until now, a lot of works on holographic RG flows in three and four dimensional gauged supergravities have appeared, see for example [10], [11], [12], [13] and [14].

A study of AdS$_6$/CFT$_5$ correspondence has not been explored in details although some works in this direction can be found in [7] and [15], see also [16] for a more recent results. To the best of the author's knowledge, only one holographic RG flow solution, studied in [17], in AdS$_6$/CFT$_5$ correspondence has been studied so far. This work will provide another example which is related to a deformation of five dimensional $N = 2$ SYM at a conformal fixed point with enhanced global symmetry $SU(2)$.

The starting point is the $F(4)$ gauged supergravity in six dimensions with $N = (1, 1)$ supersymmetries. The pure $F(4)$ gauged supergravity has been constructed in [18]. It has been known since its first construction that the scalar potential of the pure $F(4)$ gauged supergravity admits two critical points [18]. One of them is maximally supersymmetric, and the other one breaks all supersymmetries. The latter is however stable and should correspond to a non-supersymmetric conformal field theory according to the general principle of the AdS/CFT correspondence. A numerical RG flow solution interpolating between these two critical points has been given in [17]. The $F(4)$ gauged supergravity has also been studied in the context of holography in [19] in which RG flows between a UV CFT$_5$ and an IR CFT$_3$ or CFT$_2$ have been given.

In this work, we consider an RG flow in AdS$_6$/CFT$_5$ correspondence from matter coupled $F(4)$ gauged supergravity constructed in [20] and [21]. The dual CFT's with $N = 2$ supersymmetry are fixed points of the $N = 2$ $USp(2k)$ SYM theory describing the worldvolume theory of D4-branes [22], see also [23]. The fixed points corresponding to interacting CFT's arise in an infinite coupling limit. In term of brane configurations, they can be described as near horizon geometries of the D4-D8 brane system in type I$'$ theory [24]. The configuration should also be obtained in massive type IIA theory where the D8-branes are known to exist [15]. At the fixed points, the global symmetry gets enhanced to E_{N_f+1}, $N_f = 0, \ldots, 7$ with $E_1 = SU(2)$, $E_2 = SU(2) \times U(1)$, $E_3 = SU(3) \times SU(2)$, $E_4 = SU(5)$ and $E_5 = SO(10)$ [22]. $E_{6,7,8}$ are the usual exceptional groups. N_f denotes the number of flavor hypermultiplets or equivalently the number of D8-branes. The extension to theories with global symmetry $\tilde{E}_1 = U(1)$ and $E_0 = \mathbf{I}$

(no global symmetry) has been studied in [25]. The matter coupled $F(4)$ gauged supergravity whose scalars paramatrized by $\mathbb{R}^+ \times \frac{SO(4,n)}{SO(4)\times SO(n)}$ coset space can give rise to gravity duals of these 5-dimensional CFT with enhanced global symmetry by gauging the $SU(2)_R \times G$ subgroup of $SO(4) \times SO(n)$. n denotes the number of matter multiplets, vector multiplets in this case. $SU(2)_R$ is the R-symmetry given by the diagonal subgroup of $SU(2) \times SU(2) \sim SO(4)$, and G is identified with the flavor symmetry E_{N_f+1} with $n = \dim G$.

The full symmetry at the fixed points is given by $F(4) \times G$. The $F(4)$ is the superconformal group containing $SU(2)_R \times SO(5,2)$ as its bosonic subgroup. The correspondence between $F(4)$ gauged supergravity and these five dimensional CFT's has been discussed in [15], and some relations between supergravity fields with the global symmetry G being E_7 and their dual operators have been identified. These have been verified to be consistent with the spectrum of the matter coupled $F(4)$ gauged supergravity theory in [20].

We will consider a simple case of the matter coupled $F(4)$ gauged supergravity in which there are three matter (vector) multiplets. This case corresponds to $N_f = 0$ or, equivalently, no D8-branes present. We will study the scalar potential of the theory and identify some of the critical points. This gives the first non-trivial critical point in matter coupled $F(4)$ gauged supergravity and describes a new non-supersymmetric fixed point of the dual field theory in five dimensions. We then check the stability of the resulting critical point by computing the scalar mass spectrum and show that all of the scalar masses are above the BF bound.

We will also study an RG flow solution corresponding to a non-supersymmetric deformation of the UV $N = 2$ CFT with global $SU(2)$ symmetry. Since the associated RG flow is non-supersymmetric, we need to solve the full second order equations of motion for scalar fields and the metric. This makes analytic solutions extremely difficult to find. We rather give a numerical solution for the flow solution. Similar solutions in four, five and seven dimensional gauged supergravities have been given in [26], [27] and [28].

The paper is organized as follow. In section 2, we give some formulae in the matter coupled $F(4)$ gauged supergravity which forms a framework of the whole paper. We will give our notations and conventions which are slightly different from [20] and [21]. In section 3, we study the scalar potential and identify new critical points of the matter coupled $F(4)$ gauged supergravity. We will also give scalar mass spectra at each critical point. An RG flow solution connecting the trivial critical point with maximal supersymmetry and a new non-supersymmetric critical point will be given in section 4. Finally, we give our conclusions and comments in section 5.

2 Matter coupled $F(4)$ gauged supergravity

In this section, we review the construction and relevant formulae for the matter coupled $F(4)$ gauged supergravity. Although we will work with the metric signature $(-+++++)$, most of the notations and conventions used in this work will be closely parallel to those given in [20] and [21]. We refer the reader to these references for a beautiful geometric construction. The theory can be obtained from a truncation of the maximal gauged supergravity in six dimensions. Moreover, the gaugings can be described in a $O(1,1) \times SO(4,n)$ covariant form using the embedding tensor formalism [29]. However, in this paper, we will restrict ourselves to the construction of [20] and [21].

2.1 General matter coupled $F(4)$ gauged supergravity

The $F(4)$ supergravity is a half maximal, $N = (1,1)$, supergravity in six dimensions. The field content of the supergravity multiplet is given by

$$\left(e^a_\mu, \psi^A_\mu, A^\alpha_\mu, B_{\mu\nu}, \chi^A, \sigma\right)$$

where e^a_μ, χ^A and ψ^A_μ denote the graviton, the spin $\frac{1}{2}$ field and the gravitino, respectively. Bothχ^A and ψ^A_μ are the eight-component pseudo-Majorana spinor with indices $A, B = 1, 2$ referring to the fundamental representation of the $SU(2)_R$ R-symmetry. The remaining fields are given by the dilaton σ, four vectors A^α_μ, $\alpha = 0, 1, 2, 3$, and a two form fields $B_{\mu\nu}$. As in [20], it is more convenient in the gauged theory to decompose the α index into $\alpha = (0, r)$ in which $r = 1, 2, 3$.

The matter field in the $N = (1,1)$ theory is given by the vector multiplet whose field content is

$$(A_\mu, \lambda_A, \phi^\alpha).$$

We will label each matter multiplet by an index $I = 1, \ldots, n$. The $4n$ scalars $\phi^{\alpha I}$ are described by a symmetric space $SO(4,n)/SO(4) \times SO(n)$. Furthermore, the dilaton σ can be regarded as living in the coset space $\mathbb{R}^+ \sim O(1,1)$ which is the duality group of the pure supergravity theory. Together, the $4n+1$ scalars of the matter coupled theory live in the coset space

$$\mathbb{R}^+ \times \frac{SO(4,n)}{SO(4) \times SO(n)}, \tag{1}$$

and the global symmetry is given by $O(1,1) \times SO(4,n)$. The $SU(2)_R$ R-symmetry is the diagonal subgroup of $SU(2) \times SU(2) \sim SO(4) \subset SO(4) \times SO(n)$.

$O(1,1)$ can be parametrized by e^σ while the $SO(4,n)/SO(4) \times SO(n)$ coset will be parametrized by the coset representative $L^\Lambda{}_\Sigma$, $\Lambda, \Sigma = 0, \ldots, 3+n$. It is also convenient to split the $L^\Lambda{}_\Sigma$ into $(L^\Lambda{}_\alpha, L^\Lambda{}_I)$ and further to $(L^\Lambda{}_0, L^\Lambda{}_r, L^\Lambda{}_I)$. The composite connections Q and the vielbein of the coset P are defined schematically via

$$L^{-1}dL = Q + P\,. \tag{2}$$

We now come to gaugings of the matter coupled theory and will restrict ourselves to compact gaugings. The gauge group is a subgroup of $SO(4) \times SO(n) \subset SO(4,n)$. In the pure gauged supergravity, the gauge group is given by $SU(2)_R$. In the matter coupled case, we consider the gauge group of the form $SU(2)_R \times G$ with $\dim G = n$. The restriction $\dim G = n$ comes from the fact that there are only n vector multiplets, or equivalently the maximum number of the gauge fields for the gauge group G is given by n. We will denote the structure constant of the full gauge group $SU(2)_R \times G$ by $f^{\Lambda}{}_{\Pi\Sigma}$ which can be split into ϵ_{rst} and C_{IJK} for $SU(2)_R$ and G, respectively.

As usual, the gauging is implemented by covariantizing all the derivatives, and to restore the supersymmetry, fermionic mass-like terms and a scalar potential are introduced. We only give some needed information to our study of the vacua and RG flows and refer the reader to [20] and [21] for more details. Moreover, the supersymmetry transformations of fermions are also modified by some shifts at first order in the gauge couplings. The direct product structure of the gauge group $SU(2)_R \times G$ leads to two coupling constants g_1 and g_2. As has been shown in [21], without the mass parameter m of the two-form field the maximally supersymmetric AdS_6 vacuum does not exist. Therefore, only the $m \neq 0$ case is relevant for the present work.

The bosonic Lagrangian under consideration here consists of the metric and scalars [21]

$$\mathcal{L} = \frac{1}{4}eR - e\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{4}eP_{I\alpha\mu}P^{I\alpha\mu} - eV \tag{3}$$

where $e = \sqrt{-g}$. The $\phi^{I\alpha}$ kinetic term is written in term of $P^{I\alpha}_\mu = P^{I\alpha}_i\partial_\mu\phi^i$, $i = 1, \ldots, 4n$. The scalar potential is given by [21]

$$\begin{aligned} V &= -e^{2\sigma}\left[\frac{1}{36}A^2 + \frac{1}{4}B^iB_i - \frac{1}{4}\left(C^I{}_tC_{It} + 4D^I{}_tD_{It}\right)\right] + m^2e^{-6\sigma}\mathcal{N}_{00} \\ &\quad -me^{-2\sigma}\left[\frac{2}{3}AL_{00} - 2B^iL_{0i}\right] \end{aligned} \tag{4}$$

where $\mathcal{N}_{00}$ is the 00 component of the scalar matrix

$$\mathcal{N}_{\Lambda\Sigma} = L_\Lambda{}^0L^{-1}_{0\Sigma} + L_\Lambda{}^iL^{-1}_{i\Sigma} - L_\Lambda{}^IL^{-1}_{I\Sigma}\,. \tag{5}$$

Various quantities appearing in the scalar potential are defined as follow

$$A = \epsilon^{rst}K_{rst}, \qquad B^i = \epsilon^{ijk}K_{jk0}, \tag{6}$$

$$C_I^{\ t} = \epsilon^{trs}K_{rIs}, \qquad D_{It} = K_{0It} \tag{7}$$

where

$$\begin{aligned} K_{rst} &= g_1\epsilon_{lmn}L^l{}_r(L^{-1})_s{}^mL^n{}_t + g_2C_{IJK}L^I{}_r(L^{-1})_s{}^JL^K{}_t, \\ K_{rs0} &= g_1\epsilon_{lmn}L^l{}_r(L^{-1})_s{}^mL^n{}_0 + g_2C_{IJK}L^I{}_r(L^{-1})_s{}^JL^K{}_0, \\ K_{rIt} &= g_1\epsilon_{lmn}L^l{}_r(L^{-1})_I{}^mL^n{}_t + g_2C_{IJK}L^I{}_r(L^{-1})_I{}^JL^K{}_t, \\ K_{0It} &= g_1\epsilon_{lmn}L^l{}_0(L^{-1})_I{}^mL^n{}_t + g_2C_{IJK}L^I{}_0(L^{-1})_I{}^JL^K{}_t\,. \end{aligned} \tag{8}$$

Another ingredient we are going to use is the supersymmetry transformations of χ^A, λ^I_A and ψ^A_μ

$$\begin{aligned}
\delta\psi_{\mu A} &= D_\mu\epsilon_A + \frac{1}{4}g_1e^\sigma\gamma_\mu\epsilon_A + \frac{1}{4}me^{-3\sigma}\gamma_\mu\epsilon_A \\
&\quad - \left[\frac{1}{24}\left(Ae^\sigma + 6me^{-3\sigma}(L^{-1})_{00}\right)\epsilon_{AB}\right. \\
&\quad \left. + \frac{i}{8}\left(B_te^\sigma - 2me^{-3\sigma}(L^{-1})_{t0}\right)\gamma^7\sigma^t_{AB}\right]\gamma_\mu\epsilon^B, \qquad (9) \\
\delta\chi_A &= \frac{1}{2}\gamma^\mu\partial_\mu\sigma\epsilon_A - \frac{1}{4}g_1e^\sigma\epsilon_A + \frac{3}{4}me^{-3\sigma}\epsilon_A + \frac{1}{24}\left[Ae^\sigma - 18me^{-3\sigma}(L^{-1})_{00}\right]\epsilon_{AB}\epsilon^B \\
&\quad - \frac{i}{8}\left[B_te^\sigma + 6me^{-3\sigma}(L^{-1})_{t0}\right]\gamma^7\sigma^t_{AB}\epsilon^B \qquad (10) \\
\delta\lambda^I_A &= -P^I_{ri}\partial_\mu\phi^i\sigma^r{}_A{}^B\gamma^\mu\epsilon_B - P^I_{0i}\partial_\mu\phi^i\gamma^7\gamma^\mu\epsilon_A + \left(2\gamma^7D^I{}_t - C^I{}_t\right)e^\sigma\sigma^t_{AB}\epsilon^B \\
&\quad - 2me^{-3\sigma}e^{-3\sigma}(L^{-1})^I{}_0\epsilon_{AB}\epsilon^B \qquad (11)
\end{aligned}$$

where $\sigma^{tC}{}_B$ are Pauli matrices, and $\epsilon_{AB} = -\epsilon_{BA}$. In the above equations, we have given only terms involving the metric and scalar fields. The space-time gamma matrices γ^μ satisfy

$$\{\gamma^a, \gamma^b\} = 2\eta^{ab}, \qquad \eta^{ab} = \mathrm{diag}(-1,1,1,1,1,1), \qquad (12)$$

and $\gamma^7 = \gamma^0\gamma^1\gamma^2\gamma^3\gamma^4\gamma^5$.

2.2 $F(4)$ gauged supergravity coupled to three vector multiplets

In this subsection, we consider the $F(4)$ gauged supergravity coupled to three matter multiplets. The gauge group is given by $SU(2)_R \times SO(3)$ with structure constants ϵ_{rst} and ϵ_{IJK}.

We begin with the $SO(4,3)/SO(4)\times SO(3)$ coset. It is convenient to parametrize the group generators by basis elements

$$(e^{xy})_{zw} = \delta_{xz}\delta_{yw}, \qquad w,x,y,z = 1,\ldots,7 \qquad (13)$$

from which we find the following generators:

$$\begin{aligned}
SO(4) &: \qquad J^{\alpha\beta} = e^{\beta+1,\alpha+1} - e^{\alpha+1,\beta+1}, \qquad \alpha,\beta = 0,1,2,3, \\
SU(2)_R &: \qquad J^{rs} = e^{s+1,r+1} - e^{r+1,s+1}, \qquad r,s = 1,2,3, \\
SO(3) &: \qquad T^{IJ} = e^{J+4,I+4} - e^{I+4,J+4}, \qquad I,J,K = 1,2,3\,. \qquad (14)
\end{aligned}$$

Non-compact generators are given by

$$Y^{\alpha I} = e^{\alpha+1,I+4} + e^{I+4,\alpha+1}\,. \qquad (15)$$

Explicit calculations show that it is extremely difficult (if possible) to compute the scalar potential on the full 12-dimensional scalar manifold $SO(4,3)/SO(4)\times SO(3)$. As in other similar works, we will employ the method introduced in [30]. The potential is computed on a particular submanifold of $SO(4,3)/SO(4)\times SO(3)$ which is invariant under some subgroup of the gauge group $SU(2)_R\times SO(3)\sim SO(3)_R\times SO(3)$. From now on, we will use $SU(2)_R$ and $SO(3)_R$, interchangeably, since the two terminologies are convenient in different contexts.

We will study the scalar potential on scalar fields invariant under $SO(3)_{\text{diag}}$, $SO(2)_{\text{diag}}$, $SO(2)_R$ and $SO(2)$. Under $SO(3)_R\times SO(3)$, the 12 scalars transform as $(\mathbf{1}+\mathbf{3},\mathbf{3})$. Under $SO(3)_{\text{diag}}\subset(SO(3)_R\times SO(3))_{\text{diag}}$, they transform as

$$(\mathbf{1}+\mathbf{3})\times\mathbf{3}=\mathbf{1}+\mathbf{3}_{\text{adj}}+\mathbf{3}+\mathbf{5} \tag{16}$$

where the $\mathbf{3}_{\text{adj}}$ denoting the adjoint representation. For convenient, the subscript adj is used to distinguish $\mathbf{3}_{\text{diag}}$ from the vector representation $\mathbf{3}$. We immediately see that there is one singlet under $SO(3)_{\text{diag}}$. It corresponds to the following generator

$$Y_s=Y^{21}+Y^{32}+Y^{43}\,. \tag{17}$$

Under $SO(2)_R\subset SO(3)_R$ with the embedding $\mathbf{3}\rightarrow\mathbf{1}+\mathbf{2}$, the 12 scalars transform as $3\times(\mathbf{1}+\mathbf{1}+\mathbf{2})$ containing six singlets given by

$$\begin{aligned}\bar{Y}_1&=Y^{11},\qquad \bar{Y}_2=Y^{12},\qquad \bar{Y}_3=Y^{13},\\ \bar{Y}_4&=Y^{21},\qquad \bar{Y}_5=Y^{22},\qquad \bar{Y}_6=Y^{23}\,.\end{aligned} \tag{18}$$

Under $SO(2)\subset SO(3)$ with the same embedding, there are four singlets in the decomposition of the 12 scalars under $SO(2)$, $4\times(\mathbf{1}+\mathbf{2})$. The singlets are given by

$$\tilde{Y}_1=Y^{13},\qquad \tilde{Y}_2=Y^{23},\qquad \tilde{Y}_3=Y^{33},\qquad \tilde{Y}_4=Y^{43}\,. \tag{19}$$

Finally, under $SO(2)_{\text{diag}}\subset(SO(2)_R\times SO(2))_{\text{diag}}$, the 12 scalars transform as

$$(\mathbf{1}+\mathbf{1}+\mathbf{2})\times(\mathbf{1}+\mathbf{2})=\mathbf{1}+\mathbf{1}+\mathbf{1}+\mathbf{2}+\mathbf{2}+\mathbf{2}+\mathbf{3} \tag{20}$$

in which there are three singlets corresponding to

$$\hat{Y}_1=Y^{13},\qquad \hat{Y}_2=Y^{23},\qquad \hat{Y}_3=Y^{31}+Y^{42}\,. \tag{21}$$

In subsequent sections, we will study the scalar potential on these submanifolds together with the associated critical points and possible RG flows.

3 Critical points of the matter coupled $F(4)$ gauged supergravity

In this section, we study some critical points of the $F(4)$ gauged supergravity coupled to three vector multiplets. We first compute the scalar potential of

the matter coupled theory and identify some critical points. We first study the potential on the submanifold invariant under $SO(3)_{\text{diag}} \subset SO(3)_R \times SO(3)$. The coset representative is given by

$$L = e^{bY_s} \tag{22}$$

where Y_s is given in (17). We find the potential

$$\begin{aligned} V &= \frac{1}{16}[\cosh(6a) - \sinh(6a)]\left[(\sinh(8a) + \cosh(8a))\left[8\left(g_1 g_2 \sinh^3(2b) + g_1^2 - g_2^2\right)\right.\right. \\ &\quad +9\cosh(2b)\left(g_1^2 + g_2^2\right) - \cosh(6b)\left(g_1^2 + g_2^2\right)\big] \\ &\quad \left. +64m(\sinh(4a) + \cosh(4a))\left(g_1 \cosh^3 b - g_2 \sinh^3 b\right) - 16m^2\right]. \end{aligned} \tag{23}$$

where a is the dilaton σ.

As shown in [20] and [21], the trivial critical point, given by setting $a = 0$ and $b = 0$, is maximally supersymmetric provided that $g_1 = 3m$ with g_2 arbitrary. We can study the scalar mass spectrum of the 13 scalars at this critical point by expanding the potential on the full 13-dimensional scalar manifold to quadratic order. This gives the following mass spectrum:

scalars	m^2L^2
$(\mathbf{1},\mathbf{1})$	-6
$(\mathbf{1},\mathbf{3})$	-4
$(\mathbf{3},\mathbf{3})$	-6

We have labeled the scalars according to their representations under $SO(3)_R \times SO(3)$ gauge symmetry of the critical point. The AdS radius is given by $L = \sqrt{-\frac{5}{V_0}} = \frac{1}{2m}$ with $V_0 = -20m^2$. This result exactly agrees with the analysis of [20] and [21]. Using the discussion given in [15], the dilaton $(\mathbf{1},\mathbf{1})$ and the $(\mathbf{3},\mathbf{3})$ scalars do not correspond to the highest component in the supermultiplet and describe non-supersymmetric deformations. On the other hand, the $(\mathbf{1},\mathbf{3})$ scalars gives rise to supersymmetric deformations since they correspond to the highest component.

We then move to the first non trivial critical point. This is given by

$$\begin{aligned} a &= \frac{1}{4}\ln\left(\frac{3m\sqrt{g_2^2 - g_1^2}}{g_1 g_2}\right), \qquad b = \frac{1}{2}\ln\left(\frac{g_2 + g_1}{g_2 - g_1}\right), \\ V_0 &= -\frac{20m^2}{3\sqrt{3}}\left(\frac{g_1^2 g_2^2}{m^2(g_2^2 - g_1^2)}\right)^{\frac{3}{4}}, \qquad L = \frac{1}{2m}\left(\frac{3m\sqrt{g_2^2 - g_1^2}}{g_1 g_2}\right)^{\frac{3}{4}}. \end{aligned} \tag{24}$$

This critical point is non-supersymmetric as can be checked by using the supersymmetry transformations given in (9), (10) and (11). In some details, the choice $b = \frac{1}{2}\ln\left(\frac{g_2+g_1}{g_2-g_1}\right)$ automatically gives $\delta\lambda^I_A = 0$. The condition from $\delta\chi_A = 0$ gives,

for $g_1 = 3m$,

$$\delta\chi_A = \xi\epsilon_A = 0, \qquad \xi = \frac{3m\left(g_2 - \sqrt{g_2^2 - 9m^2}\right)}{4g_2}\left(\frac{g_2}{g_2^2 - 9m^2}\right)^{\frac{3}{8}} \tag{25}$$

which has no solution for non-zero g_2.

It is useful to compute scalar mass spectrum at this critical point as this will give us some information about the stability of the critical point. Furthermore, since this critical point will involve in the RG flow solution in the next section, scalar masses are useful in the holographic context. It is convenient to label the 13 scalars according to their representations under the unbroken gauge symmetry $SO(3)_{\text{diag}}$. We find the following scalar masses:

scalars	m^2L^2
$\mathbf{1}$	-6
$\mathbf{1}$	24
$\mathbf{3}$	14
$\mathbf{3}_{\text{Adj}}$	0
$\mathbf{5}$	6

The first singlet in the table corresponds to the dilaton while the second one is the singlet parametrized by our coset representative (22). There are precisely three massless scalars corresponding to three Goldstone bosons of the symmetry breaking $SO(3)_R \times SO(3) \to SO(3)_{\text{diag}}$. These should give rise to three massive vector fields whose masses we have not computed. It can be seen from the table that all scalar masses are above the BF bound. Therefore, this critical point is stable.

As in the pure $F(4)$ gauged supergravity case, there is another critical point in which $a = -\frac{1}{4}\ln\frac{g_1}{m}$ and $b = 0$. But, this critical point is unstable whenever the scalars in the matter multiplets are turned on. The scalar mass spectrum is given as follow:

scalars	m^2L^2
$(\mathbf{1},\mathbf{1})$	10
$(\mathbf{1},\mathbf{3})$	0
$(\mathbf{3},\mathbf{3})$	-10

The scalars are classified according their representations under $SO(3)_R \times SO(3)$. We can see that the $(\mathbf{3},\mathbf{3})$ scalars have mass squares which are below the BF bound.

There is another critical point which can be given, numerically. We simply give its position on the scalar manifold without going into the details here. At this critical point, we find

$$\begin{aligned} a &= -\frac{1}{4}\ln\left[\frac{3\coth b}{16\cosh(2b) - 6\cosh(4b) + 38}\left[7\sinh(3b) + 5\sinh(5b)\right.\right. \\ &\quad \left.\left. +2\sinh b\left(4\sqrt{2}\cosh^2 b\sqrt{7\cosh(4b) - 5} - 7\right)\right]\right], \end{aligned} \tag{26}$$

and b is given implicitly in term of g_2 by

$$g_2 = -\frac{3m\left[3\cosh(4b) + 4\sqrt{2}\sqrt{7\cosh(4b) - 5} + 13\right]\coth b}{8\cosh(2b) - 3\cosh(4b) + 19}\,. \tag{27}$$

From (26), a is real for $-0.82395 < b < 0.82395$. We will not give the explicit expression for the cosmological constant here but only make a comment that the critical point is AdS_6 for $-0.64795 < b < 0.64795$. This critical point is also non-supersymmetric. We have not computed the scalar masses at this critical point because of its complexity. Furthermore, it is not an integral part of the paper.

We have also studied the scalar potential on other scalar submanifolds invariant under various $SO(2)$ residual gauge symmetries. With the $SO(2)_{\text{diag}}$ invariant scalars, the coset representative takes the form

$$L = e^{a_1\hat{Y}_1}e^{a_2\hat{Y}_2}e^{a_3\hat{Y}_3}\,. \tag{28}$$

We find the scalar potential

$$\begin{aligned} V &= \frac{1}{4}e^{-6\sigma}\left[e^{8\sigma}\left[\sinh^2(2a_3)\left(g_2^2\cosh(2a_1) + g_1^2\cosh(2a_2)\right)\right.\right. \\ &\quad +4g_2\sinh^2 a_3\left(2\cosh a_1\sinh a_2\cosh^2 a_3(g_2\cosh a_1\sinh a_2 - 2g_1\cosh a_2) - g_2\right) \\ &\quad \left.-4g_1^2\cosh^2 a_3\right] - 16me^{4\sigma}\left(g_1\cosh a_1\cosh a_2\cosh^2 a_3 - g_2\sinh a_2\sinh^2 a_3\right) \\ &\quad \left.+4m^2\left(\sinh^2 a_1\cosh(2a_2) + \cosh^2 a_1\right)\right]\,. \end{aligned} \tag{29}$$

For $SO(2)$ invariant scalars, the coset representative is given by

$$L = e^{a_1\tilde{Y}_1}e^{a_2\tilde{Y}_2}e^{a_3\tilde{Y}_3}e^{a_4\tilde{Y}_4} \tag{30}$$

giving rise to the following potential

$$\begin{aligned} V &= -4g_1me^{-2\sigma}\cosh a_1\cosh a_2\cosh a_3\cosh a_4 - g_1^2e^{2\sigma} \\ &\quad +\frac{1}{4}m^2e^{-2(a_1+3\sigma)}\left[\left(e^{2a_1} - 1\right)^2\cosh^2 a_2\left[\cosh^2 a_3\cosh(2a_4) + \sinh^2 a_3\right]\right. \\ &\quad \left.+e^{4a_1}\cosh^2 a_2 - e^{2a_1}(\cosh(2a_2) - 3) + \sinh^2 a_2 + 1\right]\,. \end{aligned} \tag{31}$$

Finally, the coset representative for $SO(2)_R$ invariance is parametrized by

$$L = e^{a_1\bar{Y}_1}e^{a_2\bar{Y}_2}e^{a_3\bar{Y}_3}e^{a_4\bar{Y}_4}e^{a_5\bar{Y}_5}e^{a_6\bar{Y}_6}, \tag{32}$$

and the corresponding potential is

$$\begin{aligned}
V &= \frac{1}{4}e^{2\sigma}\left[4g_2^2\cosh^2 a_1\cosh^2 a_4\left(\cosh a_2\sinh a_3\sinh a_5\cosh a_6-\sinh a_2\sinh a_6\right)^2\right.\\
&\quad +4g_2^2\left(\sinh a_1\cosh a_5\sinh a_6-\cosh a_1\sinh a_4\left[\cosh a_2\sinh a_3\cosh a_6\right.\right.\\
&\quad \left.\left.+\sinh a_2\sinh a_5\sinh a_6\right)\right]^2+4g_2^2\left(\cosh a_1\sinh a_2\sinh a_4\cosh a_5\right.\\
&\quad \left.-\sinh a_1\sinh a_5\right)^2-4g_1^2\cosh^2 a_4\cosh^2 a_5\cosh^2 a_6\\
&\quad +\frac{1}{4}g_1^2\left(8\cosh^2 a_4\cosh^2 a_5\cosh(2a_6)+\cosh[2(a_4-a_5)]+\cosh[2(a_4+a_5)]\right.\\
&\quad \left.+2\cosh(2a_4)+2\cosh(2a_5)-14)\right]\\
&\quad -4g_1me^{-2\sigma}\cosh a_1\cosh a_2\cosh a_3\cosh a_4\cosh a_5\cosh a_6\\
&\quad +m^2e^{-6\sigma}\left[(\cosh a_1(\cosh a_2\sinh a_3\sinh a_6+\sinh a_2\sinh a_5\cosh a_6)\right.\\
&\quad +\sinh a_1\sinh a_4\cosh a_5\cosh a_6)^2+(\cosh a_1\left(\cosh a_2\sinh a_3\cosh a_6\right.\\
&\quad \left.+\sinh a_2\sinh a_5\sinh a_6\right)+\sinh a_1\sinh a_4\cosh a_5\sinh a_6)^2\\
&\quad +\cosh^2 a_1\cosh^2 a_2\cosh^2 a_3+(\cosh a_1\sinh a_2\cosh a_5+\sinh a_1\sinh a_4\sinh a_5)^2\\
&\quad \left.+\sinh^2 a_1\cosh^2 a_4\right]. \qquad (33)
\end{aligned}$$

This potential can be further truncated to the $SO(3)_R$ invariant scalars by setting $a_4 = a_5 = a_6 = 0$. The resulting potential is then given by

$$\begin{aligned}
V &= \frac{1}{4}e^{-6\sigma}\left[-16g_1me^{4\sigma}\cosh a_1\cosh a_2\cosh a_3+m^2\left[2\cosh^2 a_1\ \times\right.\right.\\
&\quad \left.\left.\left(2\cosh^2 a_2\cosh(2a_3)+\cosh(2a_2)\right)+\cosh(2a_1)-3\right]-4g_1^2e^{8\sigma}\right]. \qquad (34)
\end{aligned}$$

This potential is interesting in the sense that if we set $\sigma = 0$, it involves only the $SO(3)_R$ singlet scalars which correspond to supersymmetric deformations [15].

However, we have not found any interesting critical points of (34) whether supersymmetric or not. Similarly, apart from $L = \mathbf{I}_{7\times7}$ and $\sigma = 0$, we are not able to figure out any critical points of the other $SO(2)$ invariant potentials given above. The analysis shows that, most probably, they might not admit non-trivial critical points at all, but a more detailed analysis is needed particularly for a very complicated potential in (33).

4 An RG flow solution

In this section, we will find a numerical RG flow solution which describes a non-supersymmetric deformation of the $N = 2$ UV CFT with global symmetry $SU(2)$. It is useful to reobtain the similar solution in pure $F(4)$ gauged supergravity studied in [17]. This simpler solution will give us a general strategy for finding a numerical solution before going to a more complicated solution with more than one active scalar. Furthermore, this is also useful for a comparison with our new solution.

We first consider pure $F(4)$ gauged supergravity in which there is only one scalar field given by the dilaton σ. The dual CFT at the maximally supersymmetric critical point is given by the 5-dimensional CFT without any global symmetry corresponding to E_0 case of [25].

We will work with the canonically normalized scalar $\phi = \sqrt{2}\sigma$. In this case, $L^\Lambda{}_\Sigma = \delta^\Lambda{}_\Sigma$, and g_2 disappears from the theory. The resulting scalar potential reduces to that of [18]

$$V = m^2 e^{-3\sqrt{2}\phi} - 4gme^{-\sqrt{2}\phi} - g^2 e^{\sqrt{2}\phi} \tag{35}$$

where we have denoted the coupling g_1 by g. There are two critical points identified in [18] long time ago. At the maximally supersymmetric critical point, we can take $\phi = 0$, and the supersymmetric AdS_6 background requires $g = 3m$. Using (9) and (10), we can easily see that the choice $\phi = 0$ and $g = 3m$ gives $\delta\psi^A_\mu = 0$ and $\delta\chi^A = 0$ provided that ϵ_A are Killing spinors on AdS_6. In our convention, at this point, the cosmological constant is given by $V_0 = V(\phi = 0) = -20m^2$. The AdS radius is given by $L = \sqrt{-\frac{5}{V_0}} = \frac{1}{2m}$. Without loss of generality, we can take $m > 0$. The non-trivial critical point is given by $\phi = -\frac{1}{2\sqrt{2}}\ln 3$, $V_0 = -12\sqrt{3}m^2$ and $L = \frac{\sqrt{5}}{2(3^{\frac{3}{4}})m}$.

We then study an RG flow solution interpolating between these two critical points. The first and second critical points are identified as the UV and IR fixed points, respectively. The central charge of the five dimensional conformal field theory is given by

$$c \sim L^4 \sim \frac{1}{V_0^2}\,. \tag{36}$$

We find the ratio of the central charges

$$\frac{c_{\text{UV}}}{c_{\text{IR}}} = \left(\frac{V_0^{(\text{IR})}}{V_0^{(\text{UV})}}\right)^2 = \frac{27}{25}\,. \tag{37}$$

We can compute scalar mass at each critical point by expanding the potential to quadratic order in scalar fluctuations. At $\phi = 0$, we find $m^2_\phi L^2_{\text{UV}} = -6$ corresponding to the dual operator of conformal dimension $\Delta = 3$ by the relation $m^2L^2 = \Delta(\Delta - 5)$. The possibility for $\Delta = 2$ is excluded since there is no bosonic operator of dimension 2 in five dimensional field theories. Although the non-trivial critical point is non-supersymmetric, it is stable as has been shown in [18]. Indeed we can also explicitly compute the scalar mass at this critical point and find $m^2_\phi L^2_{\text{IR}} = 10$ which is clearly above the BF bound $-\frac{25}{4}$.

In order to study holographic RG flows, we make a standard domain wall ansatz for the metric

$$ds^2 = e^{2A(r)}dx^2_{1,4} + dr^2 \tag{38}$$

and $\phi = \phi(r)$ with the radial coordinate r. Since the flow involves a non-supersymmetric critical point, it is a non-supersymmetric flow and describes a

non-supersymmetric deformation of the UV SCFT. We cannot use BPS equations to find the solution. Rather, we need to solve the second order equations of motion given by varying the action

$$S=\int d^6x\sqrt{-g}\left(\frac{1}{4}R-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-V\right) \tag{39}$$

where the scalar potential V is given by (35). The filed equations arising from the above action can be found for example in [31]. In any case, it is not difficult to derive all equations of motion from (39). With the above ansatz, the equations of motion become

$$\begin{aligned}
\phi''+5A'\phi'-\frac{dV}{d\phi} &= 0, && (40)\\
10A'^2-\phi'^2+2V &= 0, && (41)\\
4A''+10A'^2+\phi'^2+2V &= 0\,. && (42)
\end{aligned}$$

In these equations, $A'=\frac{dA}{dr}$ and $\phi'=\frac{d\phi}{dr}$ ect. It is well-known that only two of them are independent. We will use the first two equations in solving for the solution. Note also that these equations also implies

$$A''=-\frac{1}{2}\phi'^2\,. \tag{43}$$

There is no simple way to obtain analytic solutions to these equations. Therefore, as in [17], a numerical solution will be given here. We begin with setting $g=3m$. The boundary conditions at the UV and IR points for the scalar field are $\phi_{\text{UV}}=0$ and $\phi_{\text{IR}}=-\frac{1}{2\sqrt{2}}\ln 3=-0.3884$. The equations to be solved are given by

$$\begin{aligned}
\phi''(r)+5\phi'(r)A'(r)+3\sqrt{2}m^2e^{-3\sqrt{2}\phi(r)}-12\sqrt{2}m^2e^{-\sqrt{2}\phi(r)}+9\sqrt{2}m^2e^{\sqrt{2}\phi(r)} &= 0, && (44)\\
10A'(r)^2-\phi'(r)^2-2m^2e^{-3\sqrt{2}\phi(r)}\left(12e^{2\sqrt{2}\phi(r)}+9e^{4\sqrt{2}\phi(r)}-1\right) &= 0\,. && (45)
\end{aligned}$$

Without loss of generality, we will choose $m=1$ in the solution given below since in the above equations, m can be absorbed into the radial coordinate r by scaling $r\to mr$. By using appropriate boundary conditions and a computer program *Mathematica*, we can find a solution for ϕ as shown in Figure 1. This is the same as the solution obtained in [17], and it is clearly seen that the solution interpolates between the two critical points.

We now look at the asymptotic behavior of the solution near the critical points. At the UV point, we take $r\to\infty$ and $A(r)\to\frac{r}{L_{\text{UV}}}$ with $L_{\text{UV}}=\frac{1}{2m}$. The linearized equations of motion give

$$\phi\sim c_1e^{-\frac{3r}{L_{\text{UV}}}}+c_2e^{-\frac{2r}{L_{\text{UV}}}}\,. \tag{46}$$

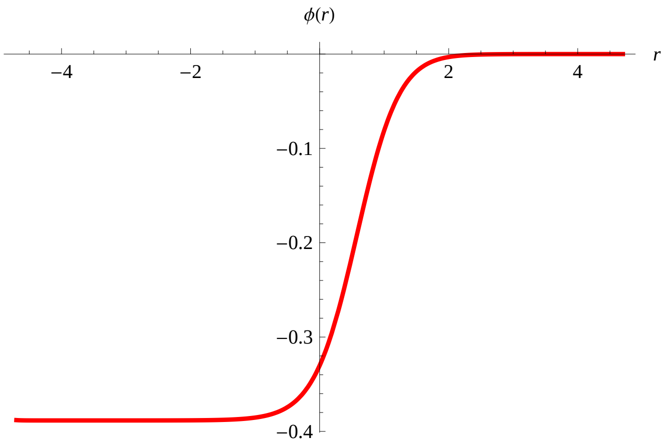

Figure 1: $\phi(r)$ solution in pure $F(4)$ gauged supergravity.

As discussed in [17], there cannot be any vev deformations since $\phi < 0$ along the flow. The physical vacuum expectation values, $\langle \mathcal{O}_\phi \rangle$, must be positive. Therefore, the scalar σ corresponds to turning on a relevant operator of dimension $\Delta = 3$ in the dual UV CFT. This is described by the non-normalizable mode $e^{-\frac{2r}{L_{\text{UV}}}}$ in (46). According to the relation between six dimensional supergravity fields and operators in the dual five dimensional field theory [15], we find that the dual operator is given by a mass term of the scalars in hypermultiplets.

At the IR point, we find

$$\phi \sim e^{\frac{\sqrt{65}+5}{2}\frac{r}{L_{\text{IR}}}} \tag{47}$$

with $L_{\text{IR}} = \frac{\sqrt{5}}{2m(3^{\frac{3}{4}})}$. We see that the corresponding operator is irrelevant and has dimension $\Delta = \frac{\sqrt{65}+5}{2} > 5$. As pointed out in [17], the dual operator acquires an anomalous dimension in the IR. We will see that this is different from the flow solution in the matter coupled theory.

We are now in a position to study a new RG flow interpolating between the maximally supersymmetric critical point and the new non-supersymmetric one found in the previous section. The procedure is the same as that of the pure $F(4)$ gauged supergravity case. We begin with equations of motion for the scalars and the metric. These are obvious generalizations to more than one scalar of the field equations (40), (41) and (42). The metric ansatz is the same as (38). The

relevant equations of motion to be solved are given by

$$a''(r)+5a'(r)A'(r)+\frac{1}{8}m^2e^{-6a(r)}\left[-32e^{4a(r)}\left(3\cosh^3 b(r)-5\sinh^3 b(r)\right)\right.$$
$$\left.+e^{8a(r)}\left(60\sinh^3(2b(r))+153\cosh(2b(r))-17\cosh(6b(r))-64\right)+24\right]=0, \quad (48)$$
$$b''(r)+5A'(r)b'(r)+8m^2e^{-2a(r)}\sinh b(r)\cosh b(r)(3\cosh b(r)$$
$$-5\sinh b(r))\left[e^{4a(r)}\sinh b(r)\cosh b(r)(5\cosh b(r)-3\sinh b(r))+1\right]=0, \quad (49)$$
$$10A'(r)^2-a'(r)^2-b'(r)^2-\frac{1}{4}m^2e^{-6a(r)}\left[32e^{4a(r)}\left(3\cosh^3 b(r)-5\sinh^3 b(r)\right)\right.$$
$$\left.+e^{8a(r)}\left(60\sinh^3(2b(r))+153\cosh(2b(r))-17\cosh(6b(r))-64\right)-8\right]=0\,. \quad (50)$$

In order to find a numerical solution, we have set $g_1=3m$ and $g_2=5m$ with the assumption of $m>0$ for positive g_2. With this choice, the UV and IR points are given by $a=b=0$ and $a=-0.05579$, $b=0.6931$, respectively. By using $m=1$ as in the previous case, we find the solutions for $a(r)$ and $b(r)$ as shown in Figure 2 and 3.

Since both $a(r)$ and $b(r)$ have the same mass at the UV point, they are dual to relevant operators of the same dimension, $\Delta=3$ in this case. The asymptotic behavior near the UV point is given by

$$a(r)\sim A_1e^{-\frac{3r}{L}}+A_2e^{-\frac{2r}{L}}, \quad (51)$$
$$b(r)\sim B_1e^{-\frac{3r}{L}}+B_2e^{-\frac{2r}{L}}\,. \quad (52)$$

The flow should again be driven by relevant operators of dimension $\Delta=3$ since there is no bosonic operator of dimension 2 in five dimensional CFT as mentioned before. Similar to the previous solution, $a(r)$ is always negative along the flow, so it cannot give rise to a vev deformation. We conclude that $a(r)$ corresponds to turning on an operator with dimension 3. Using again the relation given in [15], we find that the dual operators for $a(r)$ should be the scalar mass terms for the hypermultiplet scalars.

On the other hand, $b(r)$ is positive along the flow for our choice of $m>0$. Both g_1 and m must have the same sign, and g_2 must be positive in order to make $a(r)$ real at the IR point. The choice $m<0$ will make $g_1<0$ and also $b(r)<0$ along the flow. Then, $b(r)$ will be dual to turning on a relevant operator of dimension 3 by the same reason for $a(r)$. More precisely, at the IR point, $b(r)>0$ for $0<g_1<g_2$, and $b(r)<0$ for $-g_2<g_1<0$ with $g_2>0$. With $g_1=3m>0$, $g_2=5m>0$ as in our solution, $b(r)$ is positive along the flow. This makes it possible for $b(r)$ to give rise to a vev deformation. The solution with $B_1=0$ and $B_2=0$ corresponds to non-normalizable and normalizable modes, respectively. The flow with $B_1=0$ is driven by turning on a mass term for hypermultiplet scalars while the flow with $B_2=0$ is driven by a vev of this operator. To decide between these two possibilities, we need to compute the "superpotential" in the Hamilton-Jacobi formalism for holographic RG flows [32], [33]. The situation is

very similar to the RG flow in minimal gauged supergravity in seven dimensions studied in [26]. The similarity is even more explicit by finding the behavior of $b(r)$ near the UV point by using the "true" superpotential. This gives $b(r) \sim e^{-\frac{3r}{L_{\mathrm{UV}}}}$ which precisely corresponds to the vev of a dimension 3 operator. We will not attempt to compute the "superpotential" for the non-supersymmetric flow here due to its complexity. However, we expect that our flow is driven by turning on a source term for a dimension 3 operator corresponding to $B_1 = 0$ in (52) similar to the result of [26].

The linearized equations near the IR point give $b(r) \sim e^{\frac{3r}{L_{\mathrm{IR}}}}$. This corresponds to an irrelevant operator of dimension 8, see the discussion in [34]. We see that the operator dual to $b(r)$ acquires an anomalous dimension. This is not the case for $a(r)$ which is still dual to the operator of dimension 3 as can be seen from its mass square. This is different from the flow in pure $F(4)$ gauged supergravity. Note that since the scalar $b(r)$ from the matter multiplets has $m^2L^2 = -6$, the dual operator is given by a scalar mass term. On the other hand, if the flow had involved the $(\mathbf{1}, \mathbf{3})$ scalars with $m^2L^2 = -4$ or $\Delta = 4$, the dual operator would be a fermionic mass term.

As a final remark, we compute the ratio of the central charges by

$$\frac{c_{\mathrm{UV}}}{c_{\mathrm{IR}}} = \left(\frac{g_2^2}{g_2^2 - 9m^2}\right)^{\frac{3}{2}} \tag{53}$$

for $g_1 = 3m$. This is greater than one for $g_2 > 3m$ as required by the reality condition on a and b at the IR point. This is in agreement with the c-theorem which we expect to be true as a general result in holographic RG flows involving only scalars and the metric [2].

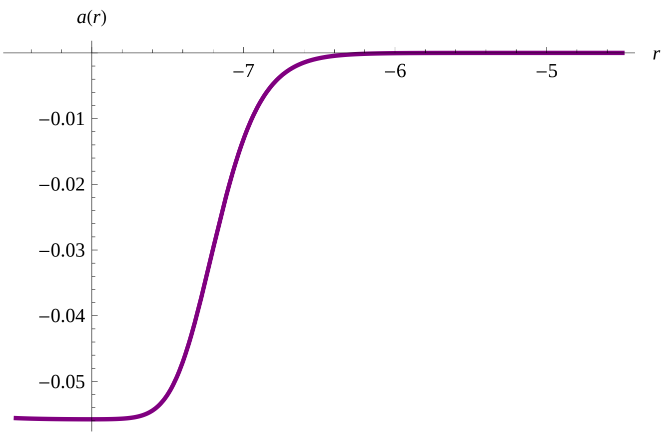

Figure 2: $a(r)$ solution in matter coupled $F(4)$ gauged supergravity.

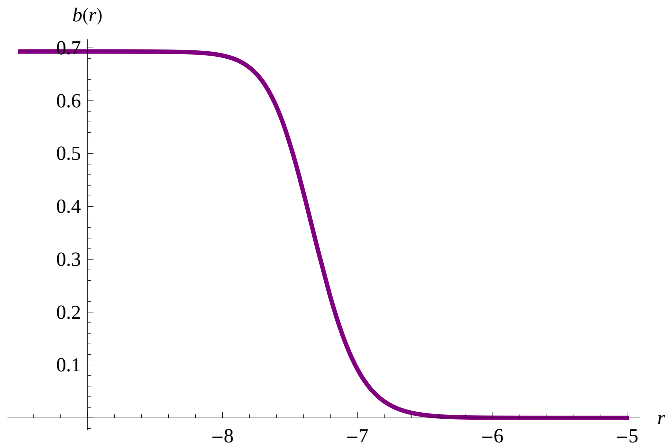

Figure 3: $b(r)$ solution in matter coupled $F(4)$ gauged supergravity.

5 Conclusions

In this paper, we have studied the scalar potential of $F(4)$ gauged supergravity coupled to three vector multiplets and the corresponding critical points. We have found new critical points apart from the known trivial one with maximal supersymmetry. One of these non-supersymmetric critical points is stable with $SO(3)$ symmetry and can be considered as a gravity dual to a non-supersymmetric CFT_5. This should be an IR fixed point of the $N = 2$ CFT_5 with symmetry $F(4)\times SO(3)$. We have also given a numerical solution describing the RG flow between these two critical points. This flow involves two active scalars corresponding to relevant operators of dimension 3 in the dual field theory. The two scalars transform under $SU(2)_R \times SO(3)$ as $(\mathbf{1}, \mathbf{1})$ and a combination of $(\mathbf{3}, \mathbf{3})$ representations. We have identified these operators with hyper-scalar mass terms. We have proposed that the flow is driven by turning on source terms for these operators. It is nice to check this proposal by computing the associated "superpotential". This will also prove that the solution is stable. We leave this issue for future works. Furthermore, we have shown that the non-supersymmetric critical point of the pure $F(4)$ gauged supergravity is unstable in the matter coupled theory.

It is interesting to study the matter coupled $F(4)$ gauged supergravity with other global symmetry group E_{N_f+1} in more details. This could give some insights to the dynamics of the D4-brane worldvolume theory in the presence of D8-branes. Furthermore, it would be useful to uplift the flow solution to ten dimensions in the context of massive type IIA theory in which the $F(4)$ gauged supergravity can be embedded [35]. The solution should also be uplifted to type II theory by the truncation given in [36]. Although the solution given here is numerical, it is possible to do a brane probe computation similar to the analysis of [27].

It is remarkable that we have not found any non-trivial supersymmetric AdS_6 critical points. The detailed analysis of the potential studied in this paper

shows that beside the trivial critical point, there are no supersymmetric critical points at least on the chosen scalar submanifolds considered here. This is in agreement with the recent result [37] on supersymmetric AdS_6 solutions from massive type IIA supergravity. In this reference, the scarcity of supersymmetric AdS_6 solutions has been pointed out. To have a definite conclusion, we need to study the matter coupled $F(4)$ gauged supergravity in more details with possibly different gauge groups to see whether there are any possibilities of supersymmetric AdS_6 critical points. Moreover, there is another possibility to obtain supersymmetric AdS_6 by using Hopf T-duality, see a discussion in [38], in type IIB theory [39].

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